IMPROVED-INVARIANT-EDGE MOMENTS WITHOUT OBJECT-EDGE TRACING

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ABSTRACT

It is pointed out the fact that after discretizing the Chen's Improved (Edge) moments line integral, it is no more necessary to maintain the restriction of computing it by edge-tracing the contour over which the integral is to be evaluated, as the original algorithm states. In practice, walking along the object contour (edge-tracing) and simply sweeping the image space, where the object is, yields exactly the same numerical results. The chain code representation of the shape contour suggested by Chen in order to evaluate the improved moments is not necessary at all. Also in this work the pattern recognition power of the improved edge moments is assessed against that of the original Hu's massive moments, it is concluded that the former are more advantageous than the latter.

RESUMEN

Se hace notar el hecho de que después de discretizar la integral de línea de los Momentos (de Borde) Mejorados propuestos por Chen, no es necesario, tal como el algoritmo lo establece, mantener la restricción de computarlo trazando los bordes del contorno del objeto sobre el cual se realiza la integración de línea. En la practica, "caminando" a través de su contorno y simplemente barriendo el espacio de la imagen en el que se encuentra el objeto, produce los mismos resultados. La representación de la forma del contorno del objeto mediante el Código de Cadena para evaluar los Momentos Mejorados, es absolutamente innecesaria. Además, en este trabajo, la potencia de reconocimiento de patrones de los momentos mejorados es evaluada teniendo como referencia los momentos masivos de Hu, se concluye que los primeros son más ventajosos que los últimos.

Key-words: Cybernetic Vision, Computer Vision, Pattern Recognition, Moment Invariants, Improved Moments, Line-integral discretization.

I. INTRODUCTION

The Moment invariants as a means to carry out pattern recognition of objects were introduced in the early sixties by [Hu, 1961], [Hu, 1962] these "traditional" massive invariants need the coordinates of all the pixels in the body and boundary of the object to be evaluated. After Hu, several papers have appeared dealing with shortcut ways to compute those massive moments [Li & cheng, 1991], [Fu, et al.,1993]. In 1993 [Chen, 1993] introduced the Improved Moment Invariants, this is a reformulation of Hu's moments and they are a set of invariants devised in such a way as to be evaluated with the object boundary (edge) pixels only. Chen suggests using a chain code representation of the shape contour in order to evaluate the improved moments.

The fact is that in the case of the Chen's improved moments it is not necessary to perform the summations in any particular order, as stated by [Montenegro, 1994], this means that it is not required any chain code representation of the object contour (edge-tracing).

Once the line integral has been discretized, it becomes a summation, and this can be evaluated in any order, for instance by sweeping the image-space top-down and left-right and considering the object pixels as they are met. After boundary extraction, edge-tracing algorithms involve a searching around every pixel neighborhood, thus they are much more expensive in computer resources than simple image-space sweeping once the boundary has been extracted.

Montenegro, [Montenegro, 1994], has measured the recognition power of Chen's improved moments by comparing it with that of the original Hu's massive moments. The evaluation of the massive moments was carried-out as usual by simply sweeping the image space and taking the object pixels as these were met. For the evaluation of the improved moments, after object edge-detection two methods were applied. In the first, the boundary pixels were considered while walking along the object boundary, this is by edge-tracing; in the second method, the image space was swept and the object boundary pixels were considered as they were met, this is, in no particular sequence.

II. HU'S ORIGINAL MASSIVE INVARIANT MOMENTS

The two-dimensional traditional Geometric Moments of order p + q of a density distribution (intensity function) f(x,y) are defined as:

$$m_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^p y^q \, f(x,y) \, dx \, dy, \quad p,q = 0,1,2,... \qquad (1)$$

and they are not invariant. The double integrals are considered over the whole area of the object including its boundary. When the geometrical moments m_{pq} are referred to the object centroid (x_c, y_c) they become the Central Moments, μ_{pq}, translation invariant:

$$\mu_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - x_c)^p \, (y - y_c)^q \, f(x,y) \, dx dy \qquad (2)$$

where: $x_c = m_{10} / m_{00}$ and $y_c = m_{01} / m_{00}$. The total area of the object is given by m_{00}. The Central Moments μ_{pq} are normalized to turn also invariant to area scaling through the relation:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \qquad \gamma = \frac{p+q}{2} + 1 \qquad (3)$$

The set of seven lowest order RTS (rotation, translation and size scaling) invariants originally deduced by Hu, is given by:

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$
$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$
$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + \\ + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$
$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12}) \\ (\eta_{21} + \eta_{03})$$
$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - \\ - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \qquad (4)$$

in practical pattern recognition situations the equations (1) and (2) are discretized for binary images according to:

$$m_{pq} = \sum_x \sum_y f(x,y) x^p y^q \qquad (5)$$

$$\mu_{pq} = \sum_x \sum_y f(x,y)(x - x_c)^p (y - y_c)^q \qquad (6)$$

Notice that m_{pq} and μ_{pq} given by equations (5) and (6) respectively can be computed by sweeping the image space.

III. CHEN'S IMPROVED MOMENTS

The Chen's improved (edge, boundary) geometrical moments are given by:

$$m_{pq} = \int_C f(x,y) \, x^p y^q dl \quad p,q = 0,1,2,... \qquad (7)$$

where the line-integral is to be evaluated along the object edge (boundary) C. Discretizing m_{pq} results in:

$$m_{pq} = \sum_{(x,y)\in C} f(x,y) \, x^p y^q \qquad (8)$$

The coordinates of the object Centroid (x_c, y_c) are given by:

$$(x_c, y_c) = \frac{1}{m_{00}} (m_{10}, m_{01}) \qquad (9)$$

In this case the length of the curve C (edge or boundary of the object) is given by $\mathbf{m_{00}}$. The boundary central moments --invariant to translation-- are given by:

$$\mu_{pq} = \int_C f(x,y) \, (x - x_c)^p \, (y - y_c)^q \, dl \qquad (10)$$

and the integral must be evaluated along the boundary

C of the object. In the discrete case μ_{pq} above becomes:

$$\mu_{pq} = \sum_{(x,y)\in C} f(x,y)\,(x-x_c)^p\,(y-y_c)^q \qquad (11)$$

Since a summation may be carried-out in any order, then equation (11) has no particular restriction concerning sequence and the pixel coordinates (x,y) may be considered in any order. The Chen's scaled normalized central moments are given by:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\alpha}} \quad \alpha = p+q+1 \quad p+q = 2,3,\ldots \qquad (12)$$

here μ_{00} is the length of the object boundary C. The η_{pq} are scale and translation invariant.

IV. IMPROVED-MOMENTS RECOGNITION PERFORMANCE

In order to assess the recognition performance of the improved moments, a set of eight non-hollowed computer synthesized objects were used, each one was sampled in six random RTS versions, see figure 1, where solid objects were used when evaluating the Massive Moments while edged objects were used to evaluate the Improved Edge (Boundary) Moments.

Both massive and (improved) boundary moments were computed for each sample. The massive moments were computed as usual by image-space sweeping, whereas for the improved moments two methods were used, edge-tracing, this is walking along the shape boundary, and image-space sweeping, this is a top-down and left-right scanning of the image-space.

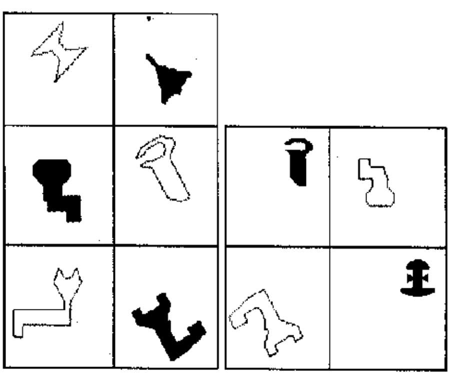

Figure 1 - RTS versions of some of the computer synthesized objects used in the experiments.

For every object and for every method investigated, the seven invariants given by equations (4) were computed for each sample, taking in each case the first one as a reference sample. The distances (similitude degree) between the reference sample and every one of the other five samples were computed with a generalization of the Pythagoras theorem to a hepta-dimensional coordinate space.

For the massive case the scaling factor was γ in equation (3), whereas for the improved moments case the factor was α in equation (12). In table (1), i-ϕ_i stands for the improved (boundary) moments, ϕ_i for the traditional (massive) moments, W appears whenever the moments refer to those computed by walking along the boundary, S appears if the moments are related to those computed by sweeping the image space.

It was experimentally found that the evaluation of the improved moments with the two methods studied produced exactly the same numerical values, this was expected because in both cases it is the very same set of pixels that is being considered and the order in which the sums are carried out does not alter the results. Table (1) displays the experimental average distances D for the 8 objects (6 samples per object) investigated. After normalization by the largest resulting value, the average distances D are:

i-ϕ_i W …… D = 1.00
i-ϕ_i S ……. D = 1.00
ϕ_i S …….. D = 0.95

where:

- i-ϕ_i : Improved (edge or boundary) mo ments.
- ϕ_i : Massive moments
- D: Average distances
- W: Walking along the object boundary.
- S: Sweeping image space

From the total average distances above it can be seen that in general it is more convenient to compute the improved moments by simply sweeping the image space instead of carrying out edge-tracing. It can also be seen from the results that the average distances in the case of boundary moments
are close enough to those corresponding to the massive moments -which are being taken as a reference for this research- this means that computing the boundary moments by sweeping the image space implies a reduction in computer time and computational

complexity and enough accurateness.

V. DISCUSSION OF RESULTS

It has been pointed out that when evaluating the improved (boundary) moments, it is not necessary to maintain the restriction of performing it by edge-tracing.

It has been experimentally found that the computation of the Chen's improved moments yield practically the same average distances between invariants as those obtained by Hu's massive moments. In this way, since the traditional massive moments are taken as a reference, it can be concluded that the Chen's boundary moments perform quite well.

This means that the boundary moments and the massive moments are practically equivalent concerning distances (similitude degrees) between objects, however in the case of the massive moments the computational complexity is $O(N^2)$ while in the boundary moments it is only $O(N)$.

Besides obtaining a reduction in computational complexity and computer time with any boundary method, if it allows image-space sweeping, like the improved moment discretized line-integral does, then the computations become even much simpler in terms of computational complexity and computer time.

In this research computer-synthesized objects were used, however, the fact that the improved moments may be computed by simple image-space sweeping instead of by object edge-tracing implies that these moments may be computed straightforwardly (without any image pre-processing) to real-life objects, which boundaries are not always flawless and fully connected.

Table 1 - Average Distances

Invariant Improved Moments, D			
Object	ϕ_i **S**	**i-**ϕ_i **S**	**i-**ϕ_i **W**
1	2.541	6.207	6.207
2	7.411	4.201	4.201
3	2.652	4.605	4.605
4	2.289	2.641	2.641
5	3.847	2.751	2.751
6	1.462	2.148	2.148
7	1.876	1.581	1.581
8	3.666	2.756	2.756
Average	**3.22**	**3.36**	**3.36**

VI. REFERENCES

Chen, C.C.. Improved Moment Invariants for Shape Discrimination. Pattern Recognition 26, 5, 683-686 (1993).

Fu, C.W., J.C. Yen, S. Chang. Calculation of moment invariants via Hadamard transform. Pattern Recognition, Vol 26, No 2, 287-294, 1993.

Hu, M.K.. Visual Pattern Recognition by Moment Invariants. IRE transactions on information theory, 179-187 (1962).

Hu,.M.K. Pattern Recognition by Moment Invariants. Proceedings of the IRE, 49, pag. 1428 (1961).

Li, B.C. - J.Chen. Fast Computation of Moment Invariants. Pattern Recognition, Vol 24, No 8. 807-813, 1991.

Montenegro, J.J. Invariant Boundary moments in Pattern Recognition. The method of C.C. Chen. Doctoral Qualification Exam (April 1994). Cybernetic Vision Research Group, Instituto de Física de Sao Carlos (IFSC), Dpto. de Física e Informática, Universidade de Sao Paulo (USP), Brazil.

OTHERS

Dudani, S., K. Breeding, R. McGhee. Aircraft identification by moment invariants. IEEE Transactions on Computers, Vol C26, No 1, Jan 1977.

Jiang, X.Y. – H. Bunke. Simple and Fast computation of moments. Pattern Recognition, Vol 24, No 8, 801-806, 1991.

Leu, J.G.. Computing a shape's moments from its boundary. Pattern Recognition, Vol 24, No 10, 949-957, 1991.

Mingfa, Z., S. Hasani, S. Bhattarai, H. Singh. Pattern recognition with moment invariants on a machine vision system. Pattern Recognition Letters, Vol 19, 175-180, April 1989

Singer, M.H.. A general approach to moment calculation for polygons and line segments. Pattern Recognition, Vol 26, No 7, 1019-1028, 1993.

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